测绘遥感信息工程国家重点实验室

State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing

Synthetic Aperture Radar Tomography – practical course

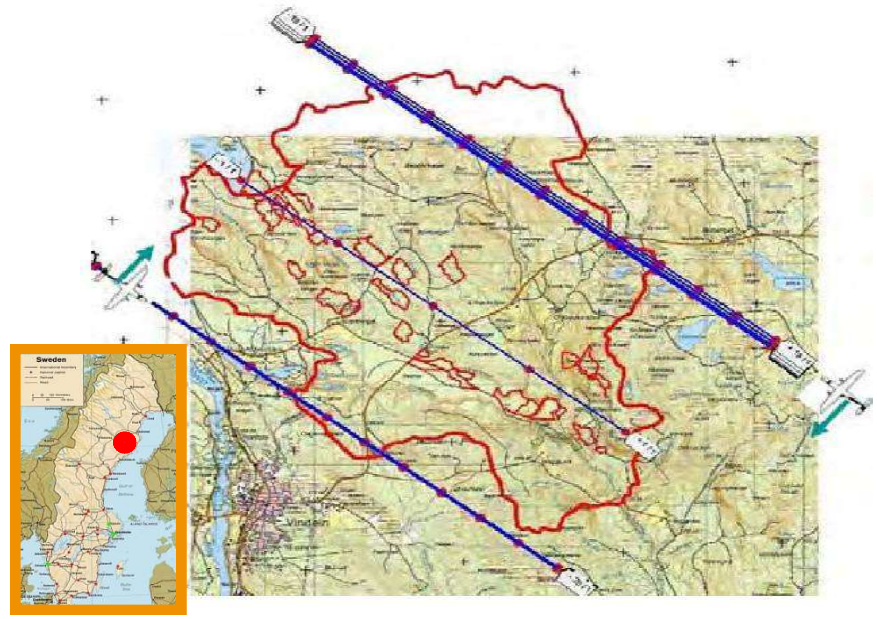
Timo Balz, Stefano Tebaldini, Laurent Ferro-Famil

TomoSAR_Main.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%
% DEMONSTRATIVE TOMOGRAPHIC SAR PROCESSING FOR FOREST ANALYSIS
% AUTHOR: STEFANO TEBALDINI, POLITECNICO DI MILANO
% EMAIL: stefano.tebaldini@polimi.it
% TEL: +390223993614
%
% THE FOLLOWING SCRIPT AND ALL RELATED SCRIPTS/FUNCTIONS AND DATA ARE INTENDED AS
% MATERIAL FOR THE TOMOSAR TRAINING COURSE HELD IN BEIJING IN FEBRUARY 2015
% BY LAURENT FERRO-FAMIL AND STEFANO TEBALDINI
%
% THIS SOFTWARE WAS DEVELOPED AND TESTED USING MATLAB R2011b
%
% SAR DATA USED IN THIS SCRIPT ARE PART OF THE SAR DATA-SET ACQUIRED BY DLR
% IN 2008 IN THE FRAME OF THE ESA CAMPAIGN BIOSAR 2008
% DATA FOCUSING, COREGISTRATION, PHASE FLATTENING, AND GENERATION OF KZ
% MAPS WERE CARRIED OUT BY DLR.
% DATA PHASE CALIBRATION WAS CARRIED OUT BY THE AUTHOR
%
% TERRAIN ELEVATION DATA USED IN THIS SCRIPT ARE EXTRACTED FROM
% THE LIDAR DATA-SET ACQUIRED BY THE SWEDISH DEFENCE RESEARCH AGENCY (FOI)
% AND HILDUR AND SVEN WINQUIST'S FOUNDATION IN THE FRAME OF THE ESA
% CAMPAIGN BIOSAR 2008
% PROCESSING OF LIDAR DATA AND PROJECTION ONTO SAR GEOMETRY WAS CARRIED OUT
% BY THE AUTHOR
%
% YOU ARE WELCOME TO ADDRESS ME QUESTIONS/COMMENTS/CORRECTIONS AT
% stefano.tebaldini@polimi.it
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%
```

Data

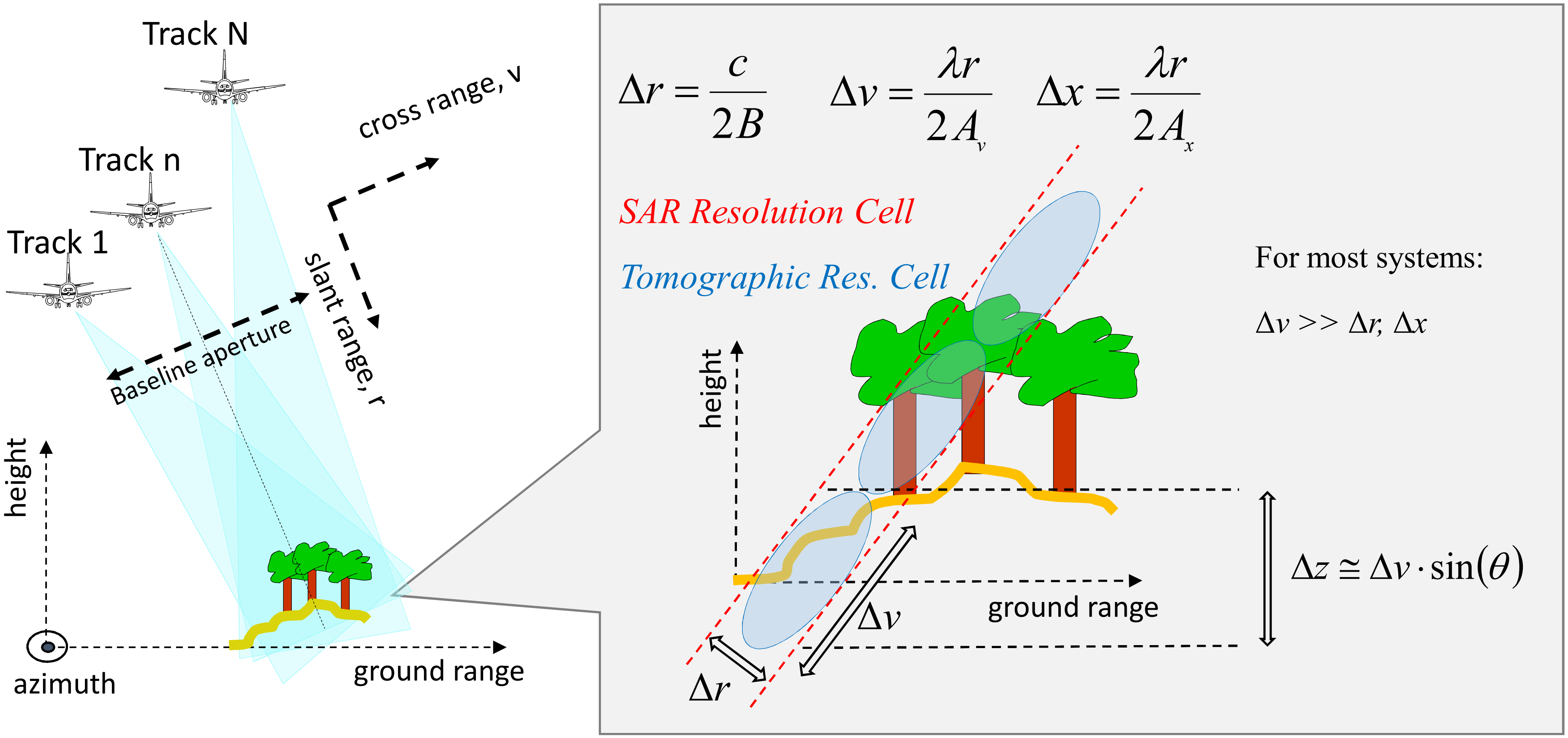
Campaign	BioSAR 2008 - ESA
System	E-SAR - DLR
Site	Krycklan river catchment, Northern Sweden
Scene	Boreal forest Pine, Spruce, Birch, Mixed stand
Topography	Hilly
Tomographic Tracks	6 + 6 – Fully Polarimetric (South-West and North-East)
Carrier Frequency	P-Band and L-Band
Slant range resolution	1.5 m
Azimuth resolution	1.6 m
Vertical resolution (P-Band)	20 m (near range) to >80 m (far range)
Vertical resolution (L-Band)	6 m (near range) to 25 m (far range)



Forward model

Resolution is determined by pulse bandwidth along the slant range direction, and by the lengths of the synthetic apertures in the azimuth and cross range directions

⇒The SAR resolution cell is split into multiple layers, according to baseline aperture



Vertical wavenumber

Each focused SLC SAR image is obtained as the Fourier Transform of the scene complex reflectivity along the cross-range coordinate

$$y_n(r,x) = \int s(r,x,v) \exp\left(-j\frac{4\pi}{\lambda r} b_n v\right) dv$$

$y_n(r,x)$: SLC pixel in the *n-th* image

$s(r,x,v)$: average complex reflectivity of the scene within the SAR 2D resolution cell at (r,x)

b_n : normal baseline for the *n-th* image

λ : carrier wavelength

Change of variable from cross range to height

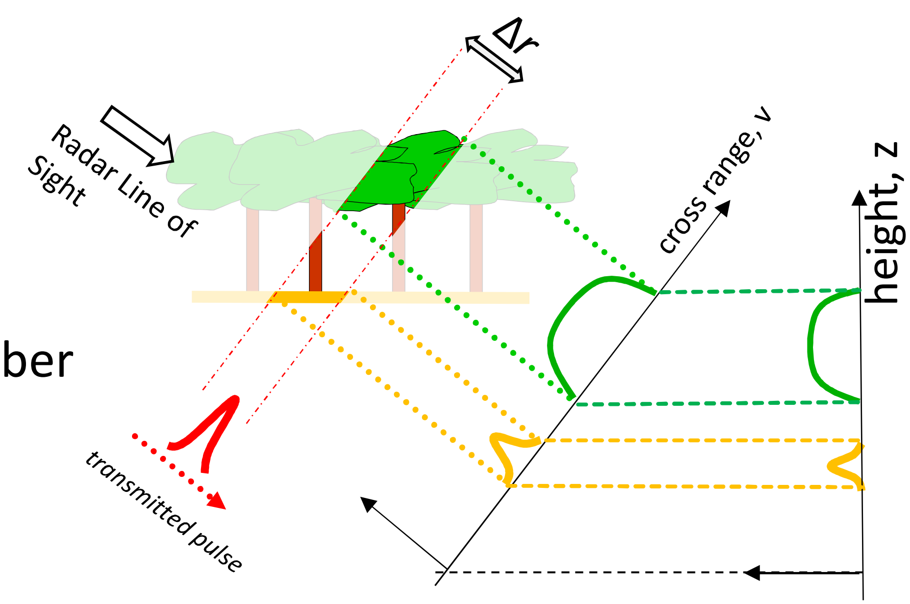
$$z = v \cdot \sin\theta$$

$$\Downarrow$$

$$y_n(r,x) = \int s(r,x,z) \exp(-jk_z(n) \cdot z) dz$$

k_z is usually referred to as vertical wavenumber or phase to height conversion factor

$$k_z(n) = \frac{4\pi}{\lambda r} \frac{b_n}{\sin\theta}$$

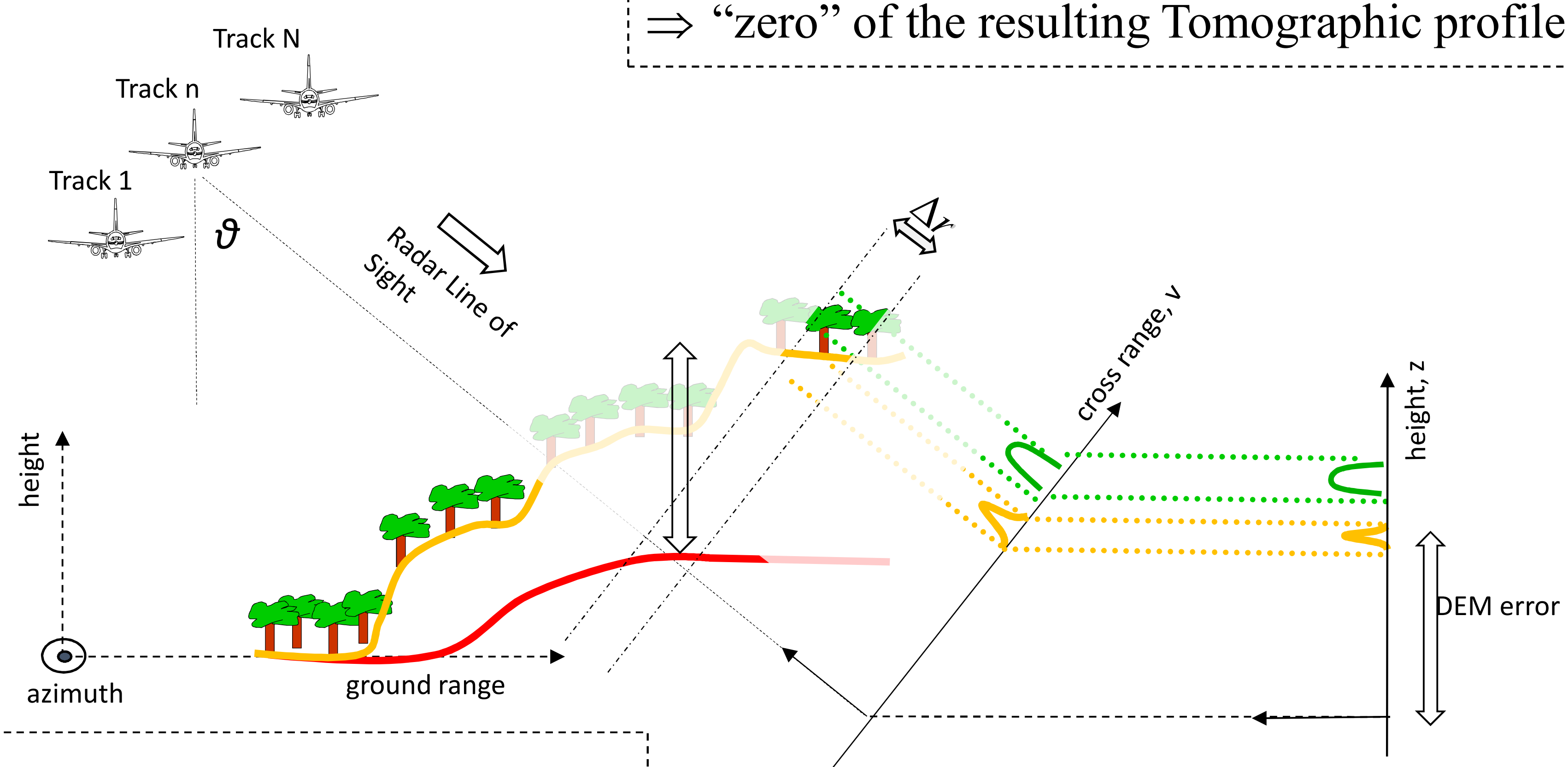


Reference height

$$y_n(r,x) = \int s(r,x,z)\exp(-jk_z(n)\cdot z)\cdot dz$$

Note: z is always intended as height with respect to a Digital Elevation Model (DEM)

$\Rightarrow$ "zero" of the resulting Tomographic profiles



Red = Reference terrain elevation

Orange = True terrain elevation

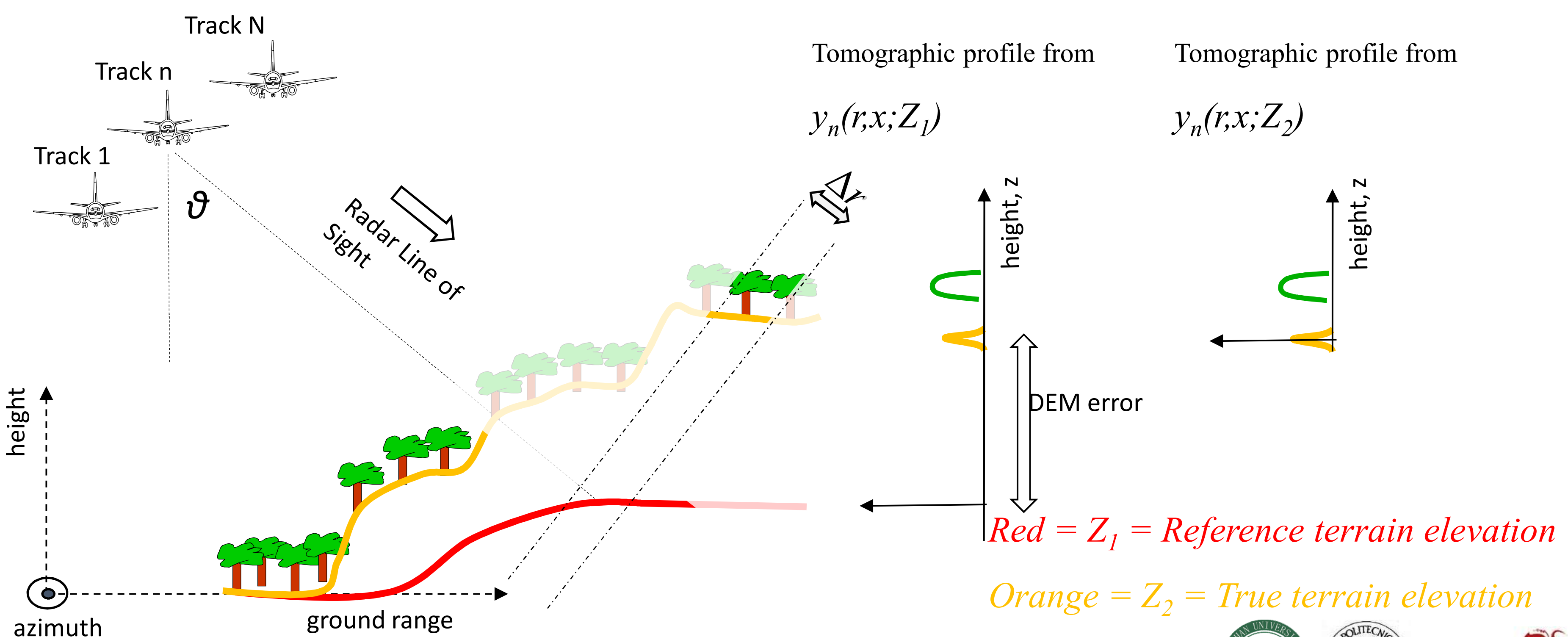
DEM subtraction

The dependence on height is limited to the phase terms $k_z z$

$$y_n(r,x) = \int s(r,x,z) \exp(-jk_z(n) \cdot z) \cdot dz$$

$\Rightarrow$ Passing from one reference DEM to another $\Leftrightarrow$ phase steering from Z_1 to Z_2

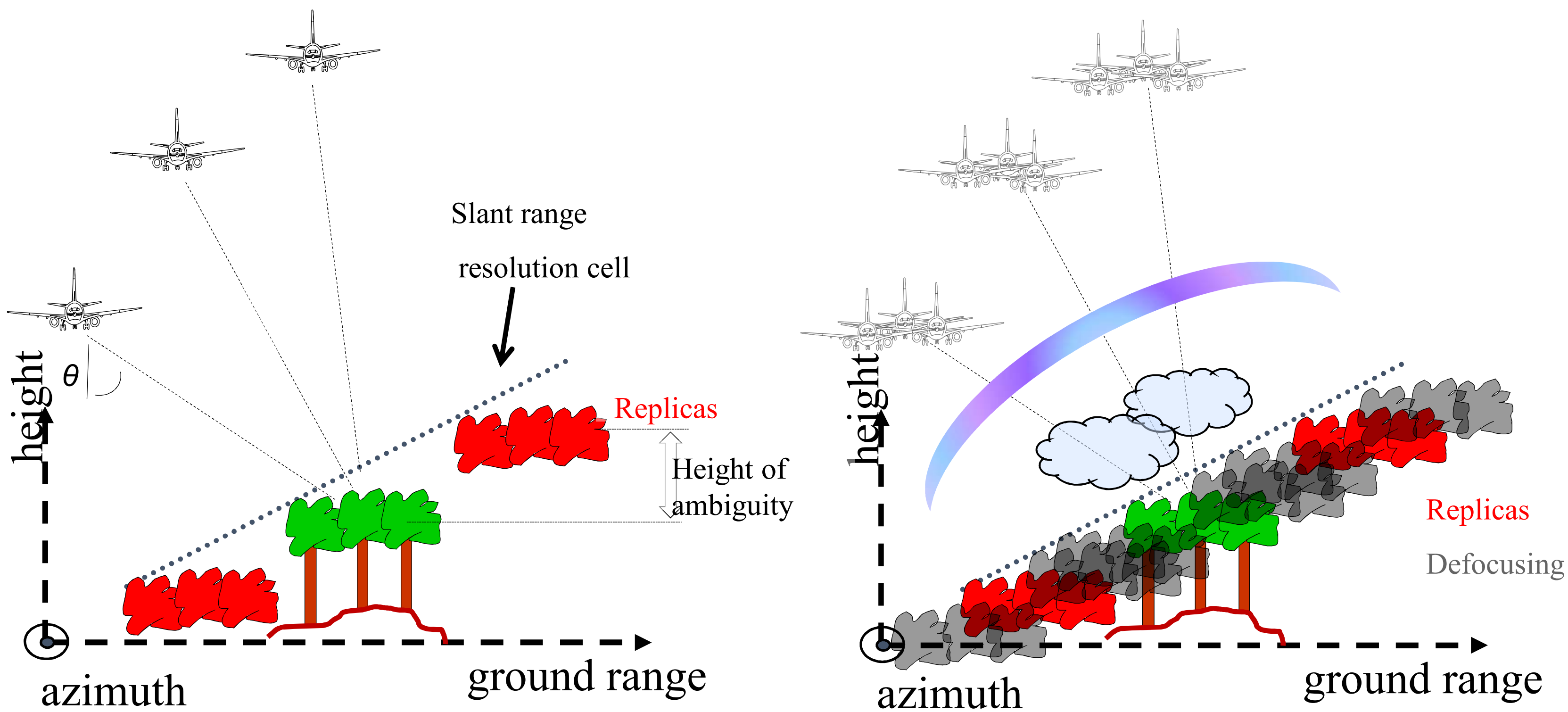
$$y_n(r,x;Z_2) = y_n(r,x;Z_1) \exp^{-jk_z(n)\cdot(Z_1 - Z_2)}$$



Phase calibration

Phase jitters in different passes result in signal defocusing

- Spaceborne: tropospheric and ionospheric phase screens
- Airborne: uncompensated platform motions *on the order of a fraction of a wavelength*

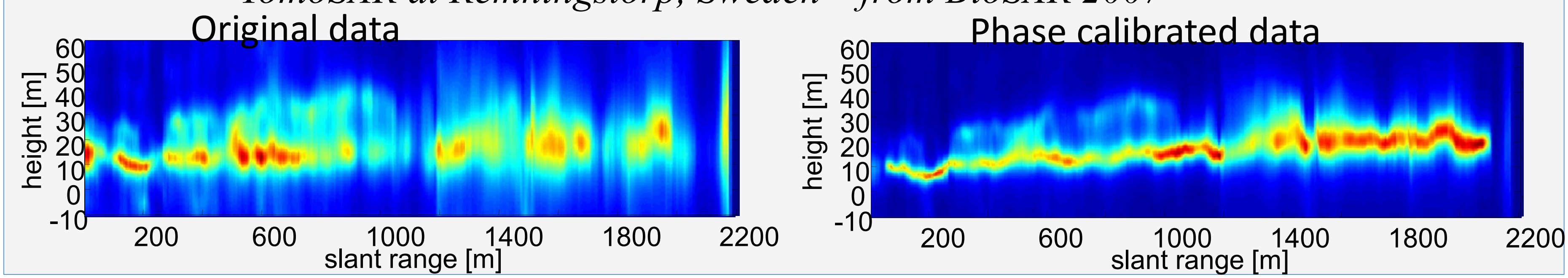


Phase calibration

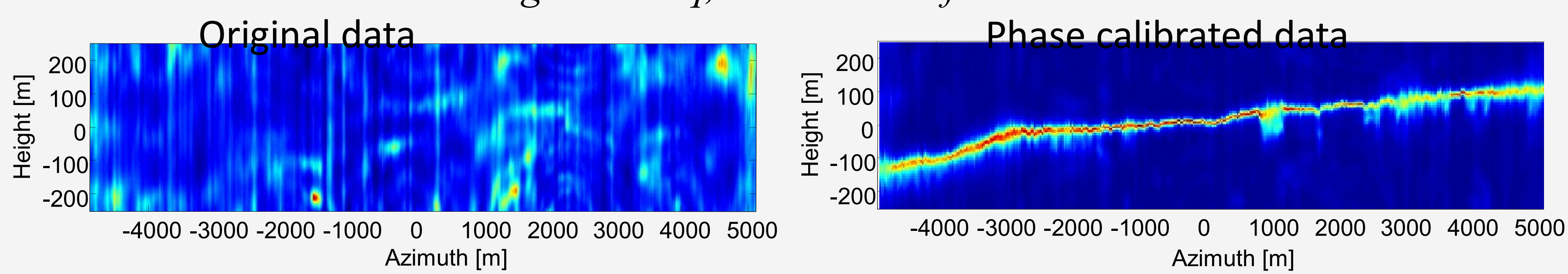
Current navigational systems employed by airborne SARs do not provide, in general, sub-wavelength accuracy concerning the location of one flight line with respect to another

→ Need for a data-driven Phase Calibration procedure

TomoSAR at Remningstorp, Sweden – from BioSAR 2007



TomoSAR at Kangerlussuaq, Greenland – from IceSAR 2012



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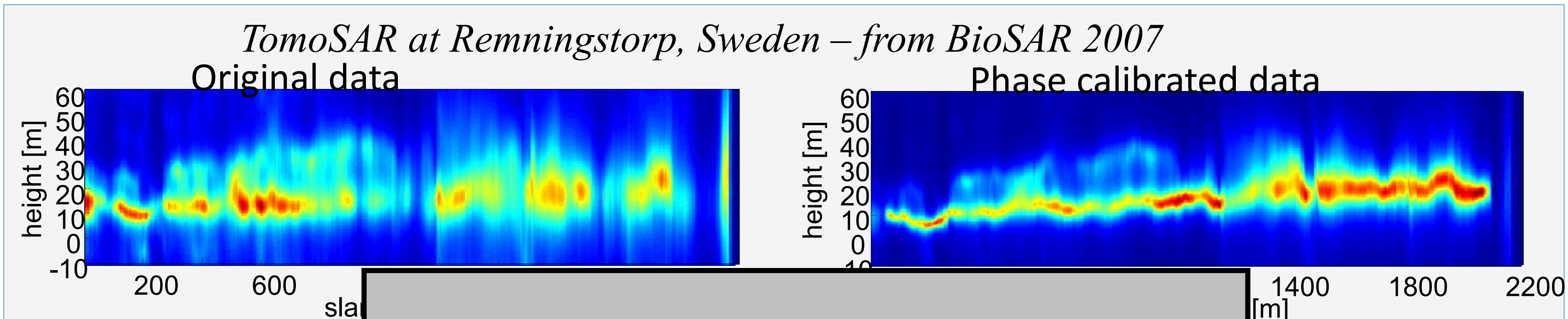
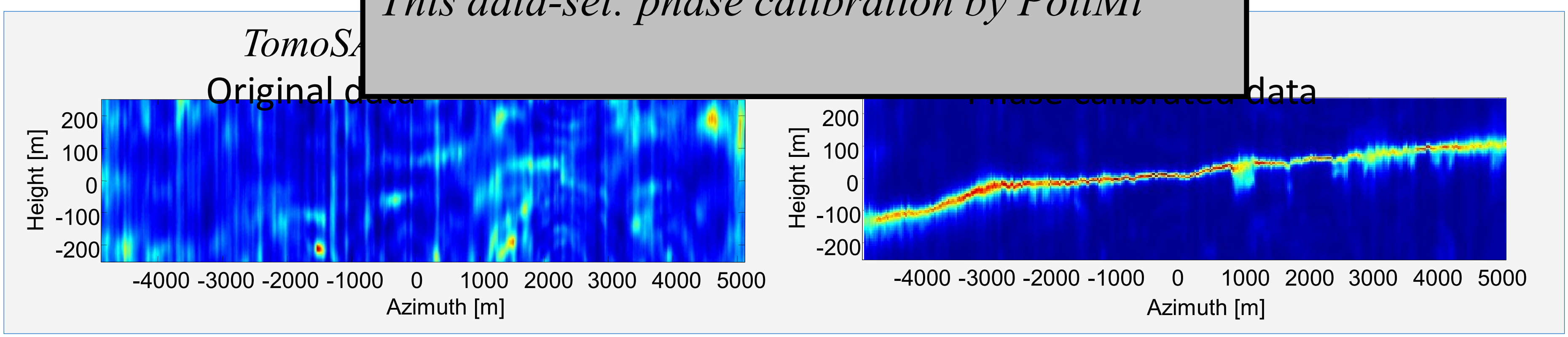
Phase calibration

Current navigational systems employed by airborne SARs do not provide, in general, sub-wavelength accuracy concerning the location of one flight line with respect to another

Need for a data-driven Phase Calibration procedure



This data-set: phase calibration by PoliMi



TomoSAR_Main.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
% LOAD DATA
if not(exist('I'))
    load('BioSAR_2_L_Band_sample_data')
    Master = 1
    [Nr,Na,N] = size(I{1})
    N_pol = length(I)
    rem_dem_flag = 1
    if rem_dem_flag % remove dem phases (optional)
        for pol = 1:N_pol
            for n = 1:N
                dem_phase = kz(:,:,n).*(DEM - DEM_avg);
                I{pol}(:,:,n) = I{pol}(:,:,n).*exp(1i*dem_phase);
            end
        end
    end
    Ch = {'HH','HV','VV'}
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
```

Notes:

Data can be referenced to a flat DEM (`DEM_avg`) or to the Lidar DEM (`DEM`)

TomoSAR_Main.m

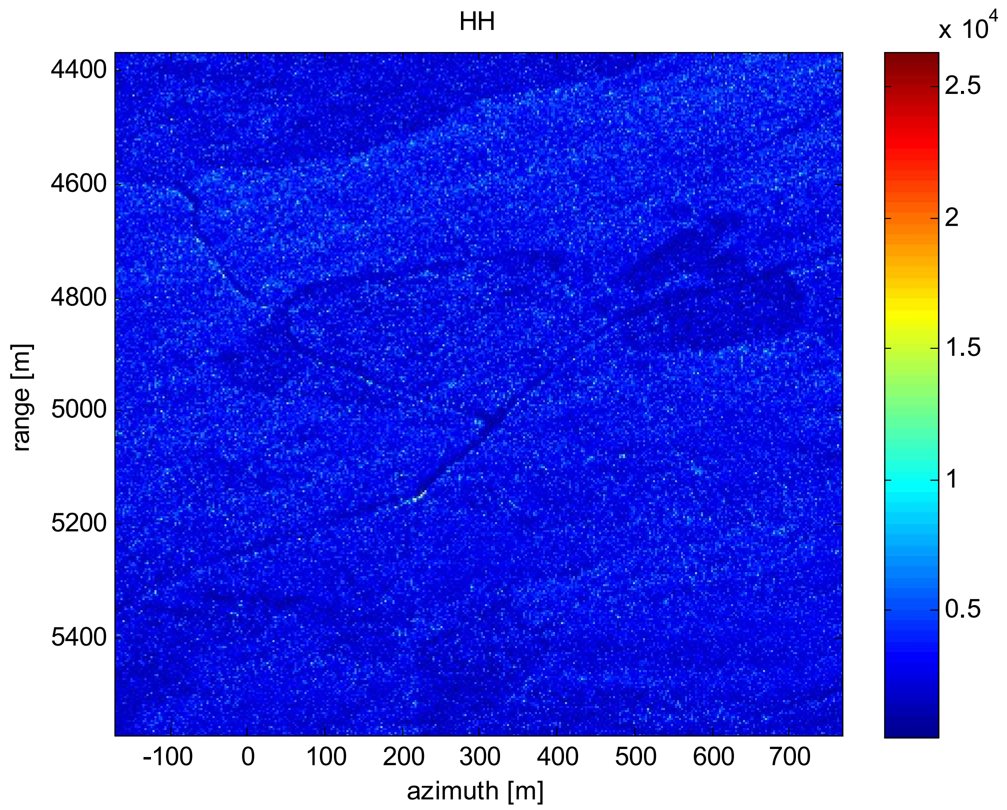
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
% Let's look at the data first....
for pol = 1:N_pol
    figure, imagesc(az_ax,rg_ax,sum(abs(I{pol}),3)), colorbar
    title(Ch{pol})
    xlabel('azimuth [m]')
    ylabel('range [m]')
end
 figure, imagesc(az_ax,rg_ax,DEM), colorbar
 title('DEM [m]')
 xlabel('azimuth [m]')
 ylabel('range [m]')
 figure, imagesc(az_ax,rg_ax,FOR_H,[0 35]), colorbar
 title('Forest height [m]')
 xlabel('azimuth [m]')
 ylabel('range [m]')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
```

Notes:

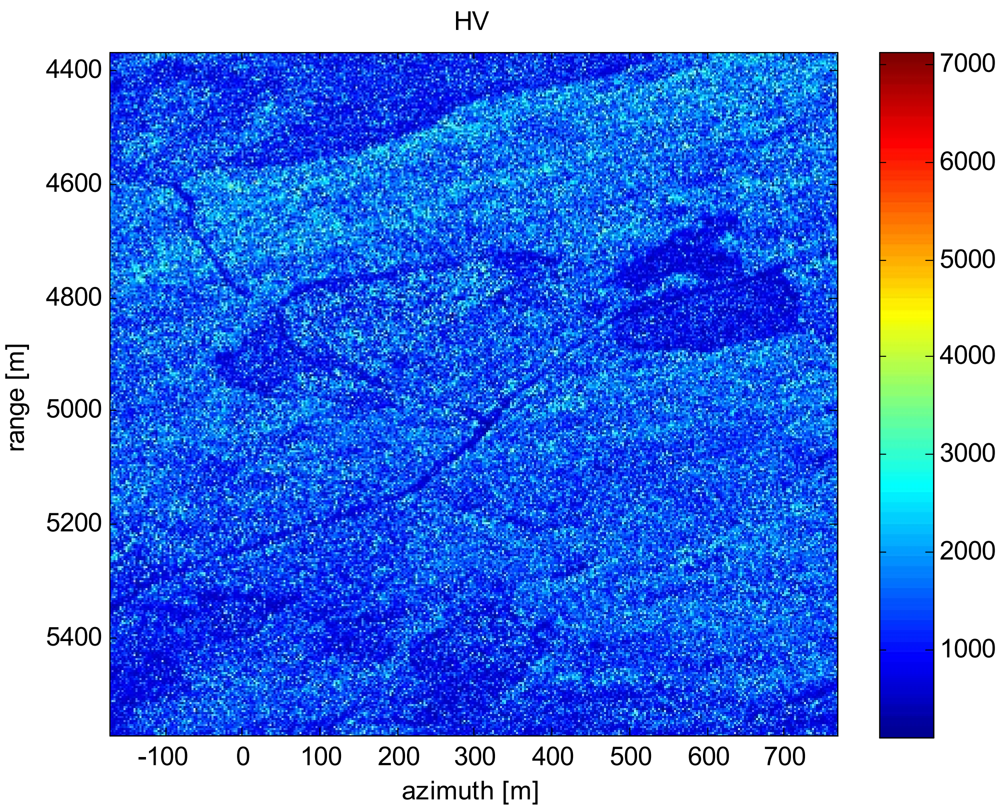
DEM = Lidar DEM

FOR_H= Lidar forest height

Results

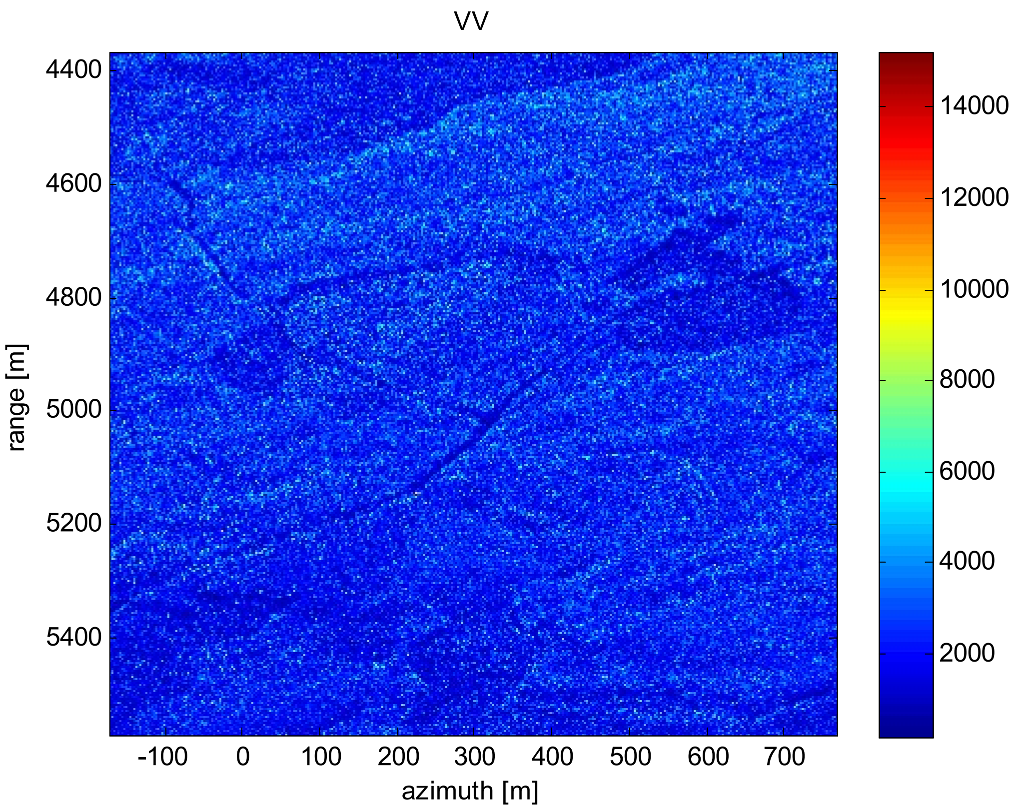

HH
x 10^4
2.5
2
1.5
1
0.5
4400
4600
4800
5000
5200
5400
range [m]
-100
0
100
200
300
400
500
600
700
azimuth [m]

Results

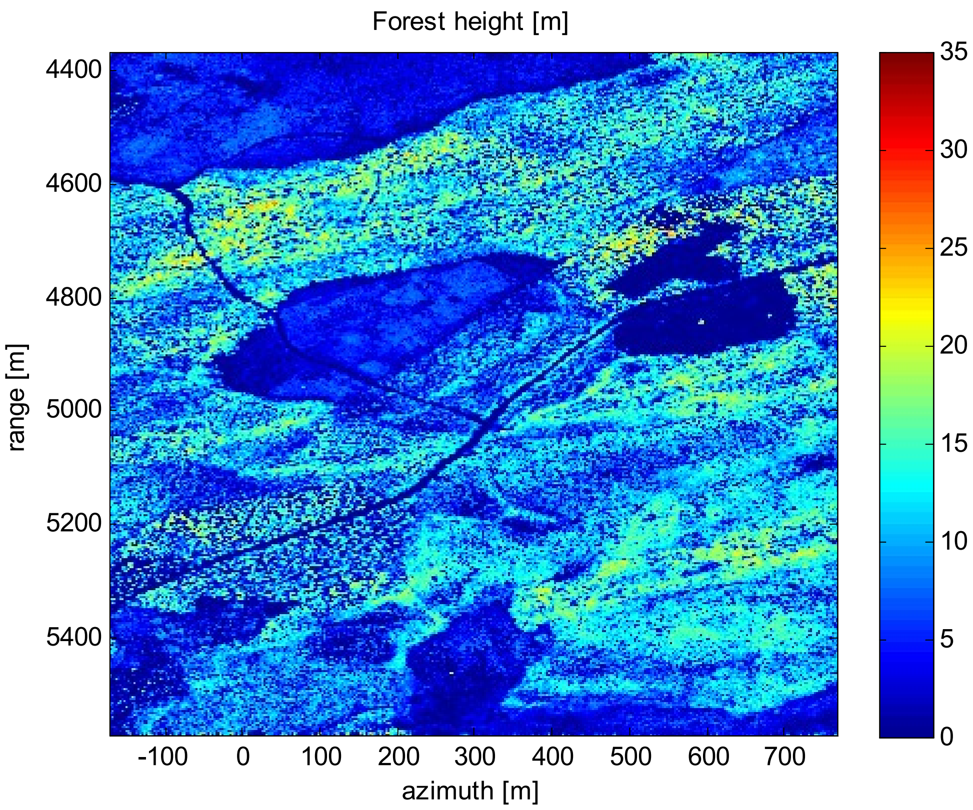


HV
4400
4600
4800
5000
5200
5400
range [m]
-100
0
100
200
300
400
500
600
700
azimuth [m]
7000
6000
5000
4000
3000
2000
1000

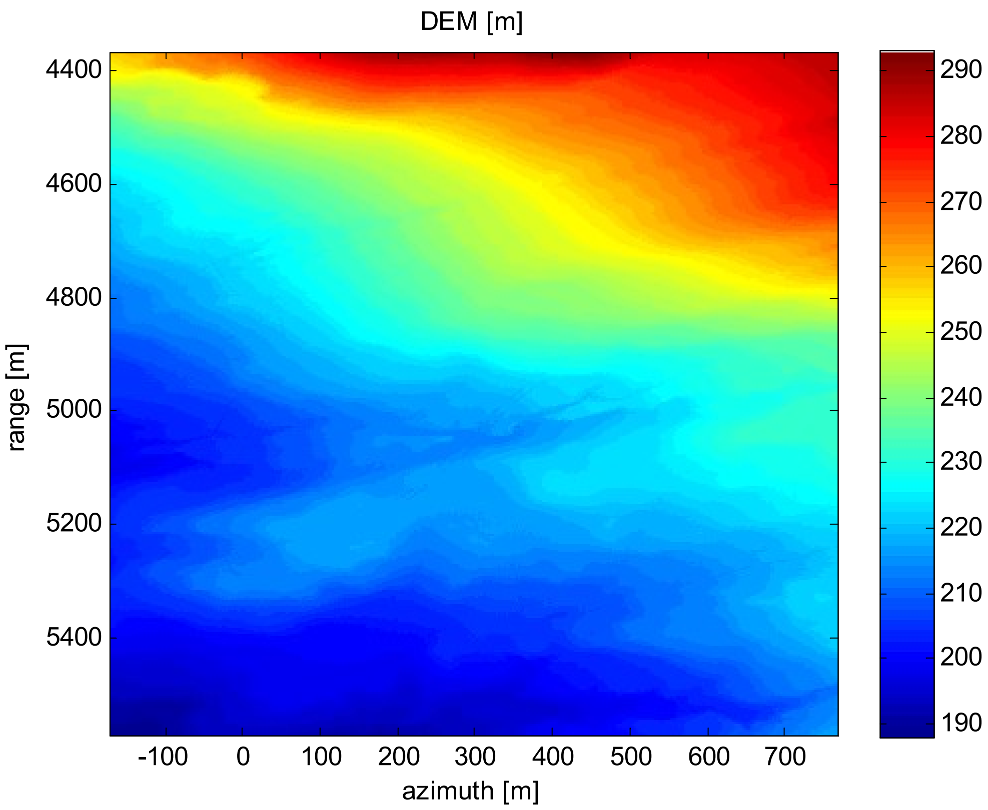
Results


VV
range [m]
4400
4600
4800
5000
5200
5400
-100
0
100
200
300
400
500
600
700
azimuth [m]
14000
12000
10000
8000
6000
4000
2000

Results


Forest height [m]
4400
4600
4800
5000
5200
5400
range [m]
-100
0
100
200
300
400
500
600
700
azimuth [m]
35
30
25
20
15
10
5
0

Results


DEM [m]
range [m]
4400
4600
4800
5000
5200
5400
-100
0
100
200
300
400
500
600
700
azimuth [m]
290
280
270
260
250
240
230
220
210
200
190

TomoSAR_Main.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
% COHERENCE EVALUATION
% estimation window (in meters)
Wa_m = 30
Wr_m = 30
[COV_4D,a_sub,r_sub] = Generate_covariance_matrix(I{1},az_ax,rg_ax,Wa_m,Wr_m);

figure, InSAR_view(abs(COV_4D),[0 1]), colorbar
title('InSAR coherences')
figure, InSAR_view(angle(COV_4D),[-pi pi]), colorbar
title('InSAR phases')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
```

Notes:

`COV_4D` *is a 4D data structure representing the complex coherence as a function of each interferometric pair, i.e.:* $y_{nm}(r,x)$

`Generate_covariance_matrix.m` = *function to evaluate COV_4D from SLC images*

`InSAR_view` = *function to view COV_4D as a big 2D matrix*

Generate_covariance_matrix.m

```
function [Cov,x_sub,y_sub] = Generate_covariance_matrix(F,x_ax,y_ax,Wx_m,Wy_m)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[Ny,Nx,N] = size(F);
% pixel sampling
dx = x_ax(2)-x_ax(1);
dy = y_ax(2)-y_ax(1);
% filter along x
Lx = round(Wx_m/2/dx);
filter_x = hamming(2*Lx+1);
% sub-sampling along x
x_sub = Lx+1:max(round(Lx/2),1):Nx-Lx;
% filter along y
Ly = round(Wy_m/2/dy);
filter_y = hamming(2*Ly+1);
% sub-sampling along y
y_sub = Ly+1:max(round(Ly/2),1):Ny-Ly;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Generate_covariance_matrix.m

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Covariance matrix evaluation
Nx_sub = length(x_sub);
Ny_sub = length(y_sub);
Cov = ones(Ny_sub,Nx_sub,N,N);
for n = 1:N
    In = F(:,:,n); % n-th image
    % second-order moment
    Cnn = filter_and_sub_sample(In.*conj(In),filter_x,filter_y,x_sub,y_sub);
    for m = n:N
        Im = F(:,:,m);
        Cmm = filter_and_sub_sample(Im.*conj(Im),filter_x,filter_y,x_sub,y_sub);
        Cnm = filter_and_sub_sample(Im.*conj(In),filter_x,filter_y,x_sub,y_sub);
        % coherence
        coe = Cnm./sqrt(Cnn.*Cmm);
        Cov(:,:,n,m) = coe;
        Cov(:,:,m,n) = conj(coe);
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

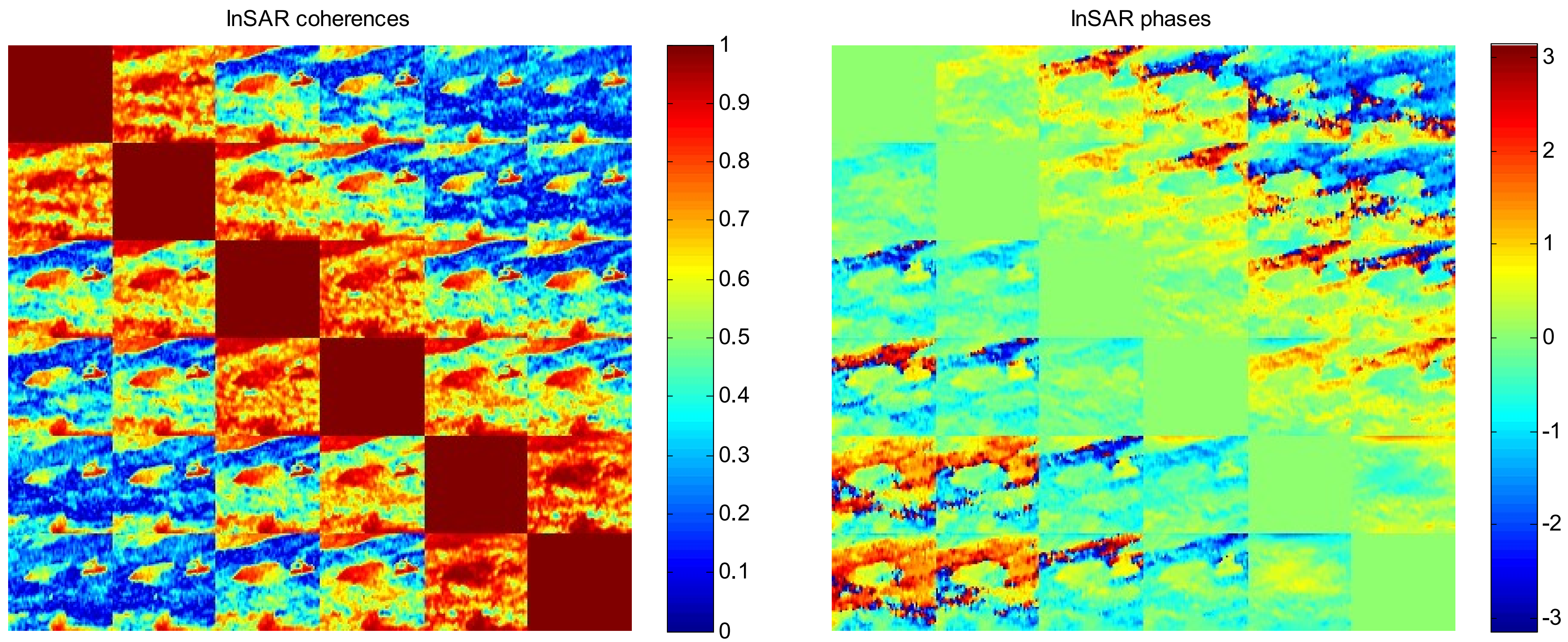
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function Cnm = filter_and_sub_sample(Cnm,filter_x,filter_y,x_sub,y_sub)
% filter and sub-sample
t = Cnm;
t = conv2(t,filter_x(:)','same');
t = t(:,x_sub);
t = conv2(t,filter_y(:),'same');
t = t(y_sub,:);
Cnm = t;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

InSar_View.m

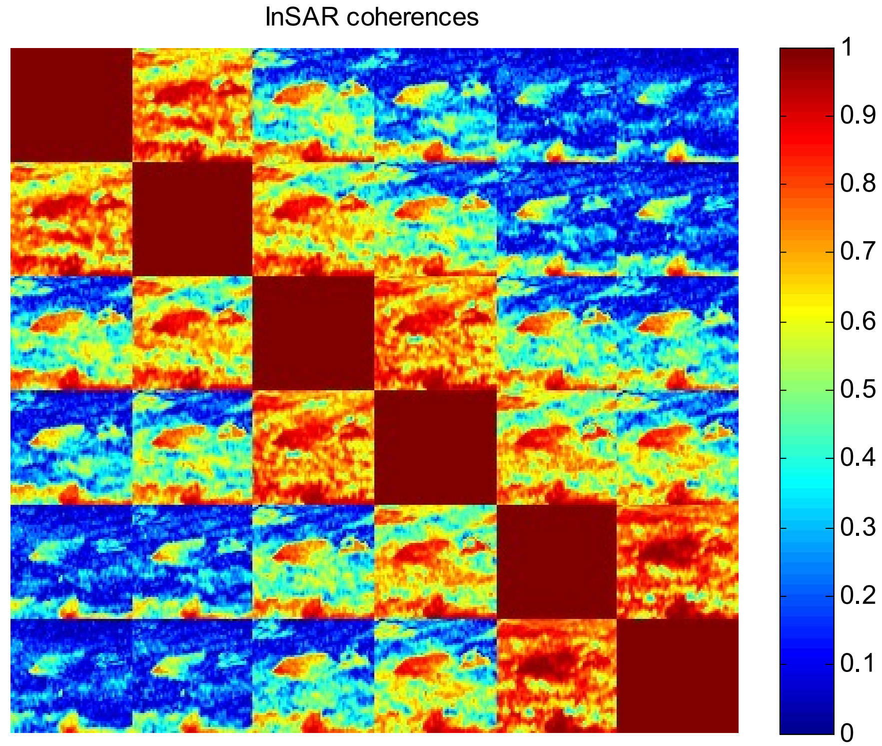
```
function InSAR_view(DX,cax)

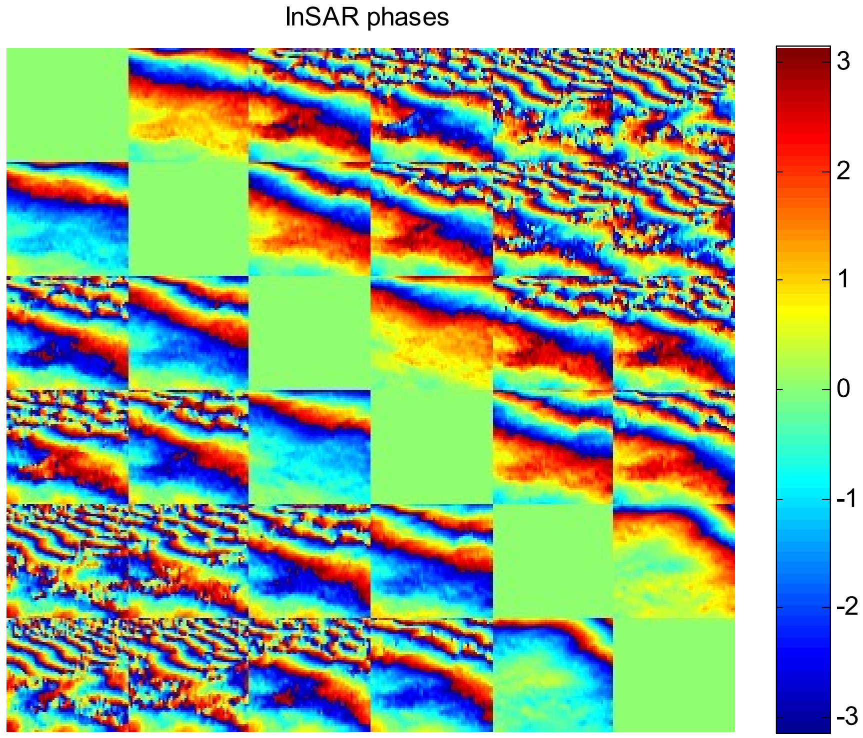
[Nx_out,Ny_out,N,a] = size(DX);
if a == N
    flag_4D = 1;
else
    flag_4D = 0;
end
DDX = zeros(N*Nx_out,N*Ny_out);
for n = 1:N
    ind_n = [1:Nx_out] + Nx_out*(n-1);
    for m = 1:N
        ind_m = [1:Ny_out] + Ny_out*(m-1);
        if flag_4D
            DDX(ind_n,ind_m) = DX(:,:,n,m);
        else
            DDX(ind_n,ind_m) = DX(:,:,m) - DX(:,:,n);
        end
    end
end
if exist('cax')==1
    if max(abs(cax-[-pi pi]))==0
    disp('phase')
    DDX = angle(exp(1i*DDX));
end
    imagesc(DDX,cax)
else
    imagesc(DDX)
end
axis off
```

Results – InSAR coherence – DEM subtracted



Results – InSAR coherence – DEM not subtracted





Notes:

Noticeable topographic phases

Lower coherence magnitudes

TomoSAR_Main.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
% TOMOGRAPHIC PROCESSING (3D focusing)
% vertical axis (in meters)
if rem_dem_flag % height w.r.t. DEM
    dz = 0.5;
    z_ax = [-20:dz:40];
else % % height w.r.t. average DEM
    dz = 1;
    z_ax = [-150:dz:150];
end
Nz = length(z_ax);
% half the number of azimuth looks to be processed
Lx = 10
% azimuth position to be processed (meters)
az_profile_m = 590;
az_profile_m = 678
az_profile_m = -92
% Focus in SAR geometry
TomoSAR_focusing
if rem_dem_flag == 0
    % the following routines have been written assuming DEM phases are removed
    return
end
% Geocode to ground geometry and compare to Lidar forest height
Geocode_TomoSAR
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
```

TomoSAR_Focusing.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% pixel index
[t,a0] = min(abs(az_ax-az_profile_m));
az_ind = a0 + [-Lx:Lx];
% Focusing
for pol = 1:N_pol
    Tomo_3D{pol} = zeros(Nz,Nr,length(az_ind));
    for z = 1:Nz
        t = I{pol}(:,az_ind,:).*exp(1i*kz(:,az_ind,:).*z_ax(z));
        Tomo_3D{pol}(z,:,:) = mean(t,3);
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Notes:

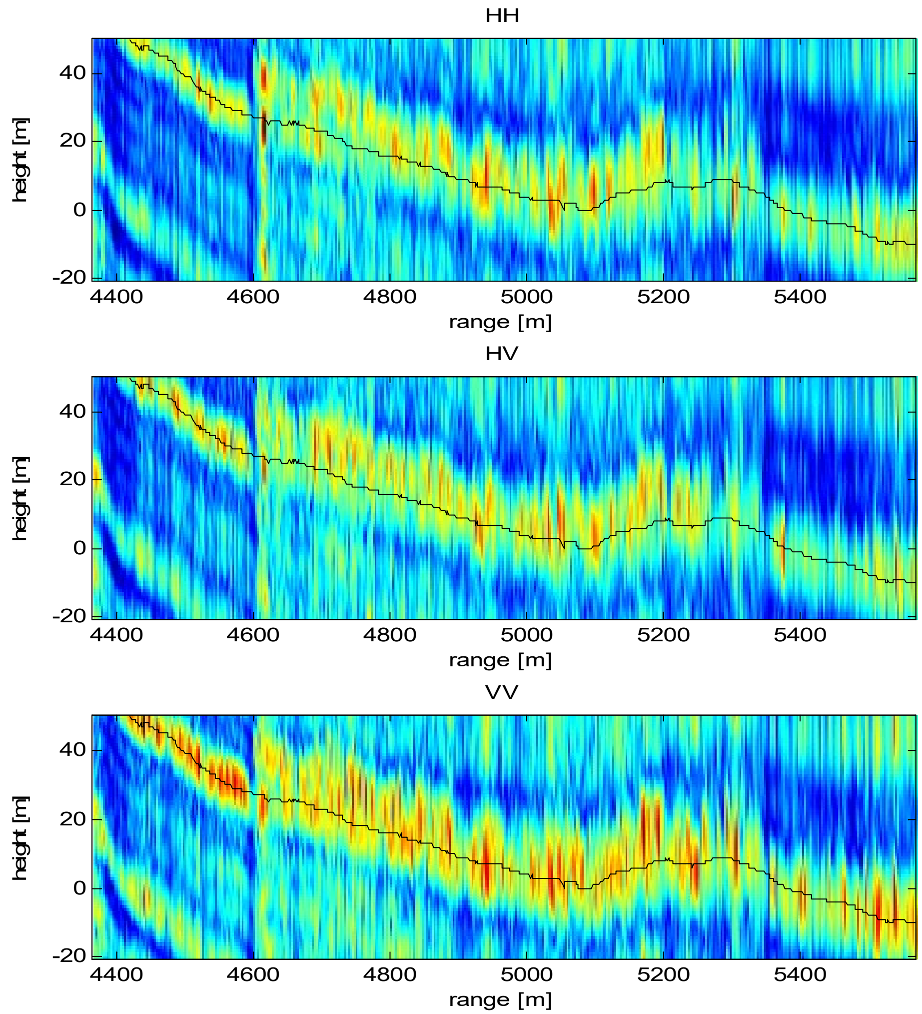
Just a discrete Fourier Transform

$$y_n(r,x) = \int s(r,x,z)\exp(-jk_z(n)\cdot z)\cdot dz \quad \Rightarrow \quad \hat{s}(r,x,z) = \sum_n y_n(r,x)\exp(jk_z(n)\cdot z)$$

Results – Tomographic Profiles

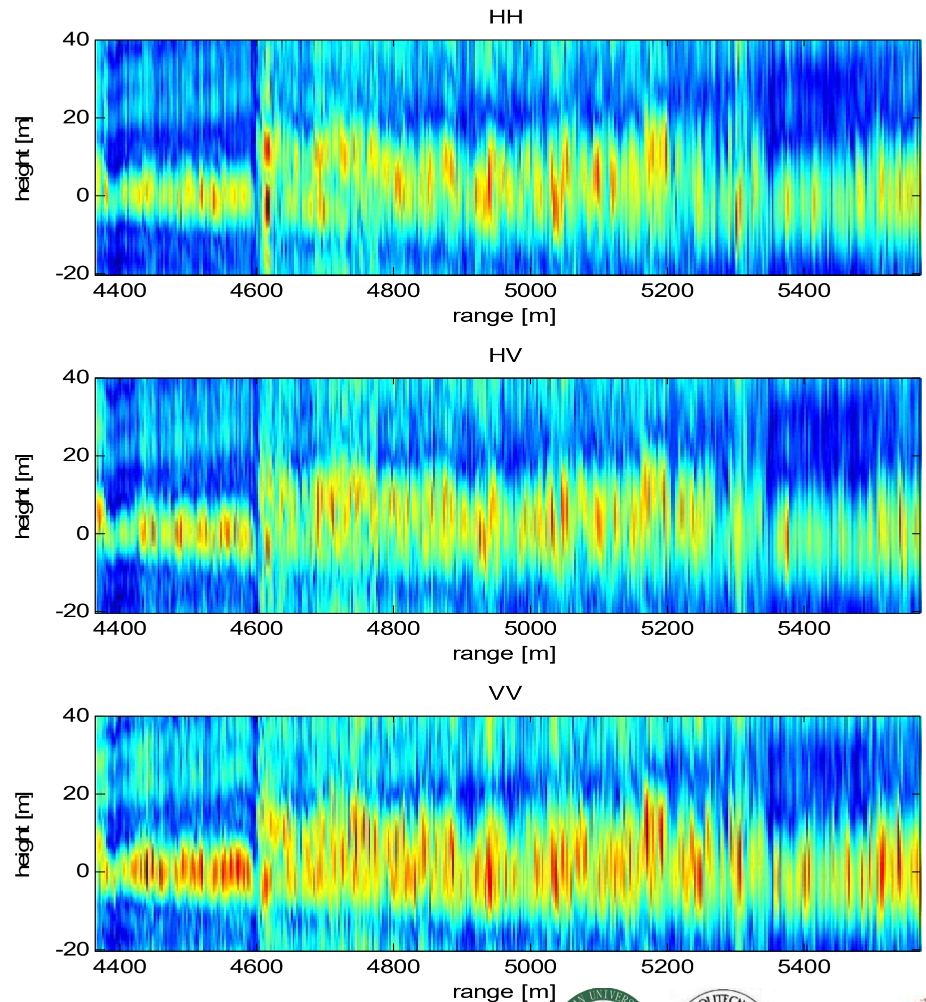
Lidar DEM not subtracted (`rem_dem_flag=0`)

⇒ *Reference height = DEM_avg = 200 m*



LIDAR DEM subtracted (`rem_dem_flag=1`)

⇒ *Reference height = LIDAR DEM*



Geocoding

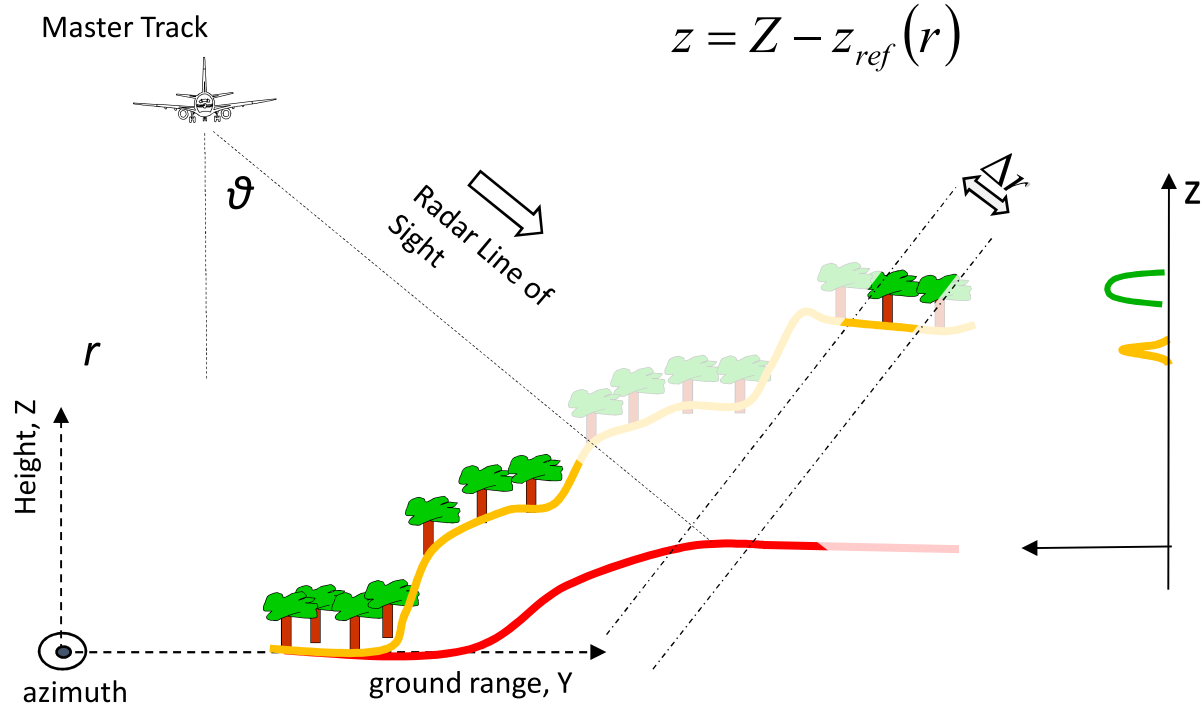
Tomographic profiles have been generated in the coordinate system (*r,z*):

- *r* = (Zero-Doppler) distance from the Master track
- *z* = height w.r.t. the reference DEM

⇒ A point at coordinates (*Y,Z*) in the ground range plane is found at

$$r = \sqrt{(Y_{Master} - Y)^2 + (Z_{Master} - Z)^2}$$

$$z = Z - z_{ref}(r)$$



Red = $z_{ref}(r)$ = Reference terrain elevation

Geocode_TomoSAR.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% pixel index
[t,a0] = min(abs(az_ax-az_profile_m));
% Master position
Sy = interp1(S{Master}.x,S{Master}.y,az_profile_m);
Sz = interp1(S{Master}.x,S{Master}.z,az_profile_m);
% Terrain elevation
dem = DEM(:,a0)';
% Forest height
for_h = FOR_H(:,a0)';
% ground range as a function of slant range
y_of_r = sqrt(rg_ax.^2 - (Sz-dem).^2) + Sy;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% absolute ground range axis
dy = 1;
y_ax_abs = [min(y_of_r)-5:dy:max(y_of_r)+5];
% absolute height axis
z_ax_abs = [min(dem)-10:dz:max(dem)+30];

% ground range as a function of slant range
y_of_r = sqrt(rg_ax.^2 - (Sz-dem).^2) + Sy;

% resample lidar dem and lidar forest height from range to ground range
dem_gr = interp1(y_of_r,dem,y_ax_abs,'linear',nan);
for_h_gr = interp1(y_of_r,for_h,y_ax_abs,'linear',nan);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Geocode_TomoSAR.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% ground range, height coordinates
[Za,Ya] = ndgrid(z_ax_abs,y_ax_abs);
% slant range
R = sqrt( (Sy-Ya).^2 + (Sz-Za).^2 );
% reference dem
Z_ref = interp1(rg_ax,dem,R,'linear','extrap');
% height w.r.t. reference dem
Z = Za - Z_ref;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Geocode tomograms
for pol = 1:3
    tomo_sar = Tomo_filt{pol};
    tomo_sar = tomo_sar./max(tomo_sar(:));
    % Geocoded tomogram
    tomo_geo = interp2(rg_ax,z_ax,tomo_sar,R,Z);

    % Geocoded tomogram - height w.r.t. Lidar
    tomo_geo(isnan(tomo_geo)) = 0;
    for y = 1:length(y_ax_abs)
        tomo_geo_rel(:,y) = interp1(z_ax_abs,tomo_geo(:,y),z_ax + dem_gr(y));
    end

          %%%%%%%%%%%%%%%%% Draw pictures here%%%%%%%%%%%%%%%%%%%%%%5

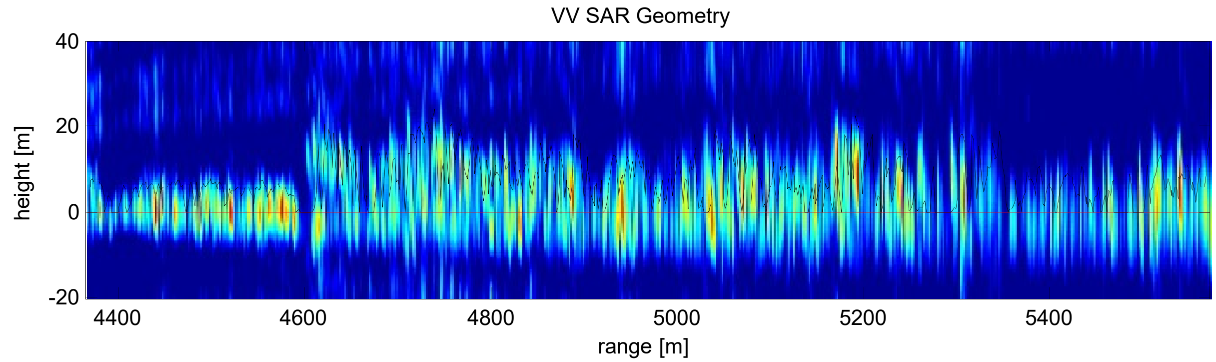
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Geocoding - Results

SAR geometry

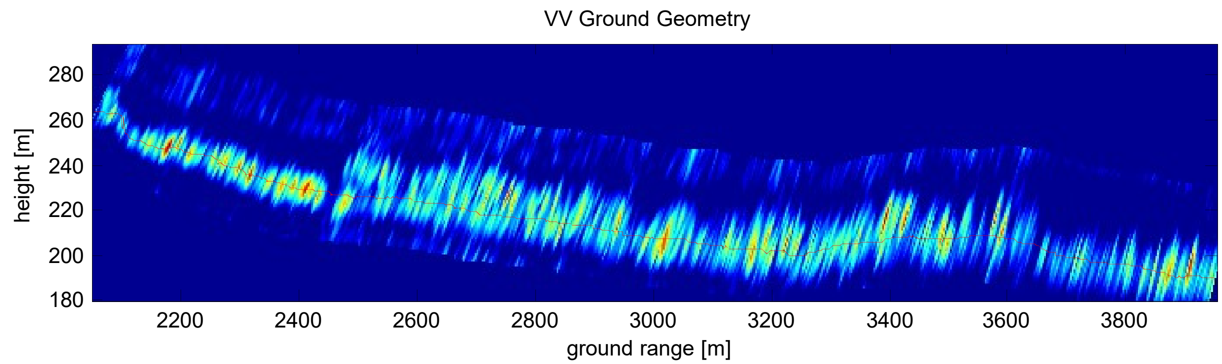
Slant range – height w.r.t. reference DEM

Note: Lidar forest height not matched



Ground geometry

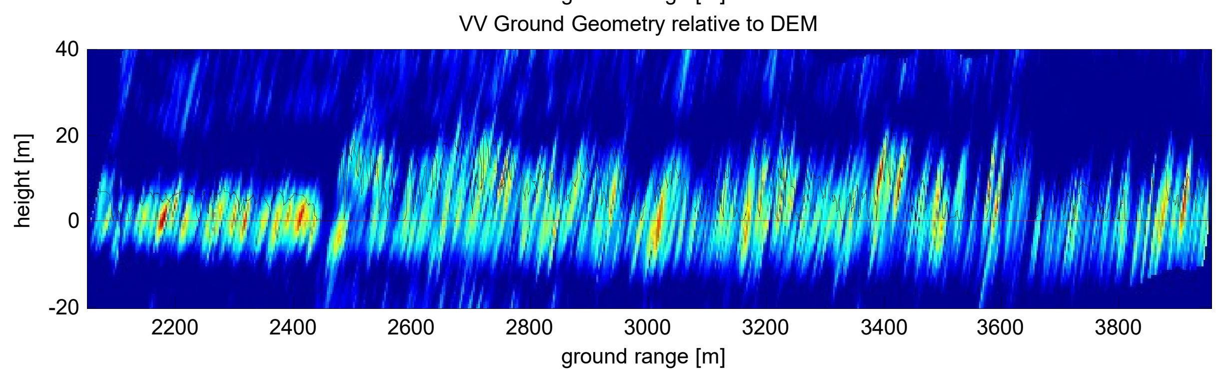
Ground range – height



Ground geometry w.r.t. reference DEM

Ground range – height w.r.t. reference DEM

Note: Lidar forest height well matched



Red = Lidar terrain *Black = Lidar forest height*

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%% THANK YOU %%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% FEEL FREE TO CONTACT ME AT:
% EMAIL: stefano.tebaldini@polimi.it
% TEL: +390223993614
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Questions?

balz@whu.edu.cn